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Measures of Social and Occupational Mobility

HUMAN populations migrate not only from one region to another but also from one occupation to a second, between socio-economic categories and likewise. Consequently, the distribution of the total population among the constituent classes changes from one generation to another or from one period of time to another. This distribution, however, will eventually reach a stable state. In the case of social mobility, these distributions obtaining in successive generations are modified by variations in the number of offsprings to different progenitors and by inter-dependences among social groups occupied by the different offsprings of the same progenitor. In the case of occupational mobility, on the other hand, variations in the number of transitions by different individuals during a unit of time have to be recognised.

Measures of social and occupational mobility based on stochastic models for representing such transitions over time and during generations have been suggested, among others, by Prais (1955), Matras (1960) and Bartholomew (1967). The present study attempts to characterise possible approaches to the measurement of mobility, to examine relative merits of the measures suggested by previous workers and lastly to provide a few new measures of social mobility. An illustrative calculation of the different measures from Glass and Hall's (1954) data has been added.

Transition Model

Let $p_{ij}^{(t)}$ denote the probability of transition from the i-th class at time (in generation) t to the j-th class at time (in generation) $t + 1$. Obviously,

$$\sum_{j=1}^{k} p_{ij}^{(t)} = 1.$$

Also, let $\pi_1^{(t)}$ $(1 = 1, 2, \ldots, k)$ denote the expected proportion of the total population at time t remaining in class 1. Then, we can specify the nature of changes as

$$\pi^{(t+1)} = [P^{(t)}]\, \pi^{(t)}, \tag{1}$$

where $P^{(t)} = (p_{ij}^{(t)})_{k \times k}$ is the transition probability matrix and $\pi^{(t)}$ is the vector giving the population distribution at time t. A repeated application of (1) gives, on the assumption that $P^{(t)} = P$ is independent of time, $\pi^{(t)} = (P)^t \pi^{(0)}$. If P is regular, P^{∞} will exist and the limiting distribution will be given by $\pi = P^{\infty} \pi^{(0)}$. This has to satisfy the relation $\pi = (P')^{\infty} \pi$ which is true if $p_{ij}^{(\infty)} = p_j$.

Measurement of Mobility

Three distinct approaches for developing measures of mobility can be enumerated. Firstly, such measures can be based on some or all elements of the transition probability matrix. There may be a few derived measures in this category which involve the limiting distribution π. In fact, most of the measures of mobility reported thus far correspond to this approach. Secondly, we may look upon the distributions $\pi^{(t)}$ and $\pi^{(t+1)}$ prevailing in two successive times as two multinomial populations and we may attempt to measure the divergence or distance between them. Lastly, for any distribution $\pi^{(t)}$ we may find out the departure from an equal distribution $\pi_0 = \left(\frac{1}{k}, \frac{1}{k} \ldots \frac{1}{k}\right)$, and measure the change in this departure during the course of a time-unit. An undesirable feature of measures obtainable from the last two approaches is their dependence on the initial distribution $\pi^{(t)}$.

To facilitate the interpretation of an observed value of any measure of social or occupational mobility, let us extend the notions of a perfectly mobile society, a perfectly immobile society and a society showing extreme movement to the general case of k classes by means of the following transition probability matrices respectively

$$P_1 = \begin{vmatrix} \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} \\ \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} \\ \cdots & \cdots & & \cdots \\ \frac{1}{k} & \frac{1}{k} & \cdots & \frac{1}{k} \end{vmatrix} \qquad P_2 = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & & \cdots & & \cdots \\ 0 & 0 & 0 & & 1 \end{vmatrix}$$

and

$$P_3 = \begin{vmatrix} 0 \; \frac{1}{k-1} & \cdots & \frac{1}{k-1} \\ \frac{1}{k-1} & 0 & \frac{1}{k-1} \\ \cdots & \cdots & \cdots \\ \frac{1}{k-1} & \frac{1}{k-1} & \cdots \; 0 \end{vmatrix}.$$

These are no doubt narrow generalisations of Prais' ideas, since the general forms of P_1 and P_3 could have been taken as

$$P_1^* = \begin{vmatrix} r_1 & r_2 & \cdots & r_k \\ r_1 & r_2 & \cdots & r_k \\ \cdots & & & \cdots \\ r_1 & r_2 & \cdots & r_k \end{vmatrix} \quad \text{and } P_3^* = \begin{vmatrix} 0 & p_{12} & \cdots & p_{1k} \\ p_{21} & 0 & \cdots & p_{2k} \\ \cdots & & & \cdots \\ p_{k1} & p_{k2} & \cdots & 0 \end{vmatrix}$$

where

$$\sum_{i=1}^{k} r_1 = \sum_{i=2}^{k} P_{1i} = \sum_{i \neq 2} p_{2i} \ldots = \sum_{i=2} p_{k_i} = 1.$$

The matrix P_1 defines in fact a special case of perfect mobility.

Matrices P_1 and P_2 are independent and hence $P_1^{\infty} = P_1$ and $P_2 = P_2^{\infty}$. The corresponding limiting (equilibrium) distributions are $(1/k, 1/k \ldots 1/k)$ and $(\pi_1^{(0)}, \pi_2^{(0)} \ldots \pi_k^{(0)})$. $P_3^{*\infty}$ does not exist and hence there is no equilibrium distribution when the society is in extreme movement. [The diagonal elements of P_3^t are equal to the off-diagonal elements of P_3^{t-1}]. It may be incidentally stated that $P_1^{*\infty} = P_1^*$ and the corresponding equilibrium distribution reached after one transition is $(r_1, r_2, \ldots, r_k)$. Thus a perfectly immobile society has a limiting distribution being the same as the initial distribution; a perfectly mobile society attains a limiting distribution defined by the transition probabilities while a society showing extreme movement never reaches an equilibrium distribution.

For a perfectly immobile society, the distance between $\pi^{(t)}$ and $\pi^{(t+1)}$ is zero and the inequality in $\pi^{(t)}$ is the same as that in $\pi^{(t+1)}$.

Previous Measures of Mobility

A simple measure of mobility based on all the elements of P is $|P|$. For $k = 2$, the values $|P|$ corresponding to cases of perfect immobility, perfect mobility and extreme movement are 1, 0 and -1 respectively. Difficulty arises for cases $k > 2$. Although for a perfectly mobile and a perfectly immobile society, $|P|$ retains the above values, confusion is created by the case of extreme movement as also by other intermediate levels of mobility. Whenever any two columns of P are identical implying perfect mobility for only two classes, $|P| = 0$. Different sets of p_{ij}, $i \neq j$, representing different patterns and levels of mobility may yield different values of $|P|$ including 0 and 1. It can be shown that if $p_{ii} = 0$ for each i and p_{ij} $(i \neq j) = 1/k-1$, $|P| = [(-1)^{k-1}/(k-1)^{k-1}]$. Also $|P| = 0$ for an extreme movement characterised by a matrix like

$$\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$$

with $k = 3$. $|P| = 1$ whenever $p_{ii} = 0$ and $p_{ij} = 0$ or 1 and these unities conform to a latin square requirement of occurring once in each row and once in each column e.g.,

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

P is non-singular and none of the classes is empty after the transition. With $p_{ii} = 0$ and $p_{ij} = 0$ or 1 but unities not conforming to the above configuration $|P| = 0$ and some of the classes are empty after the transition, e.g.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and } P \neq \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

resulting in complete withdrawal of offsprings from class 3 and class 1, respectively.

Another over-all measures of mobility is the production of the total population mobile or the complementary proportion, i.e. $1 - \sum_i p_{ii}\, \pi_i^{(t)}$ or $\sum_i p_{ii}\, \pi_i^{(t)}$. Its dependence on the distribution $\pi^{(t)}$ can be obviated by considering the limiting stable distribution π and taking $1 - \sum_i p_{ii}\, \pi_i$ or $\sum_i p_{ii}\, \pi_i$.

The proportion mobile will be equal to $1 - 1/k$ for perfect mobility and 0 for perfect immobility.

A more direct and meaningful measure is obtained by counting the class-boundaries crossed in passing from one generation to the next, i.e.,

$$D' = \sum_i \sum_j \pi_i^{(t)}\, p_{ij}\, |\, i - j \,|$$

or the corresponding measure taking into account the limiting distribution, i.e., $D = \Sigma\ \Sigma\ \pi_i\ p_{ij}\ |\ i - j\ |$, if $p_{ii} = 1$, $D = 0$. If in addition $k = 2$, $D = 1$ for a society showing extreme movement and $D = \frac{1}{2}$ for a perfectly mobile society, irrespective of the initial or the limiting distribution. Here, a measure like $D_1 = 2\,(\frac{1}{2} - D)$ will have values comparable to $|\,P\,|$.

In case of perfect immobility for any number k of classes

$$D = \frac{1}{k} \sum_{i=1}^{k} \pi_i^{(0)} \quad |\, i - i \,| = 0$$

and in case of perfect mobility

$$D = \frac{1}{k^2} \sum_{i} \underset{\neq}{} \sum_{j} \quad |\, i - j \,| = \frac{(k-1)(k+1)}{3k}$$

However, this measure is meaningful only when $| i - j |$ is meaningful, e.g., a graded social system or a system of income classes. Another disadvantage of D for $k > 2$ is that it cannot be used for comparing two societies with different groupings of the classes.

It may be contended that a better idea about transitions can be had by considering a set of measures rather than a single measure. The measures suggested by Prais (1955) are

$$\mu_j^* = \frac{1 - \pi_j}{1 - p_{jj}}$$

$j = 1, 2, \ldots, k$. For k classes $\mu_j^* = 1$ for perfect mobility and $\mu_j^* = \infty$ for perfect immobility. These measure μ_j^* depend only on the diagonal elements of P. Off-diagonal elements can be taken account of if we consider the diagonal elements of p^t for $t = 1, 2, \ldots$. Prais (1955) also gave another set of measures based on the proportion $p_{jj}^{(n)}$ of individuals originally in the j-th class who are still in that class after n units of time. A measure of mobility for the n-th generation can be taken as $P_{jj}^{(n)}/p_j$. For $n = 1$ this measure has a direct relation to the previous measure of Prais. However, for $n > 1$ it takes account of the off-diagonal elements of P.

The ratio $p_{jj}^{(n)}/p_{jj}$ assumes the value unity both for perfect mobility and perfect immobility and becomes infinite in the case of extreme movement. This makes difficult the interpretation of an observed value of this ratio.

Suggested Measures of Mobility

As a single measure based on transition probabilities, we may consider the trace of P. For a perfectly mobile, a perfectly immobile and an extremely moving society tr $P = T$ has values 1, n and 0 respectively, whatever be k. These values can be reduced to 0, 1 and -1 resyectively. We can consider

$$T' = -1 + \frac{(k^2 - 2)T + (2 - k)T^2}{k(k - 1)}.$$

Easier to calculate than $| P |$, T is based only on the diagonal elements of P. A more serious defect of T is that if $p_{ii} = 0$, $T = 0$, irrespective of the values of p_{ij} $(i \neq j)$.

We may regard the distributions of the population at times t and $t+1$ as two multinomial populations and measure the divergence between them, according to Bhattacharyya (1945-46). The vectors $\pi^{(t)}$ and $\pi^{(t+1)}$ may be considered to be the direction cosines of two straight lines through the origin in a k-dimensional space and a measure of divergence between the two populations is the square of the angle Δ^2 between these two lines, where

$$\cos \Delta = \Sigma \sqrt{\pi_i^{(t)} \pi_i^{(t+1)}}$$

Obviously, $\Delta = 0$, if $P = I$ or when the society is immobile perfectly. For other states of mobility, the numerical value of Δ depends on $\pi_i^{(t)}$. For a perfectly mobile society the value of cos Δ will be $\Sigma \sqrt{r_i \pi_i^{(0)}}$. For the special case of perfect mobility defined by P_1 cos $= 1/k \, \Sigma \sqrt{\pi_i^{(0)}}$. This will be 1 if and only if $\pi_i^{(0)} = 1/k$ for all i. In case of an extreme movement characterised by $p_{ii} = 0$ and $p_{ij} = \frac{1}{k-1}$ $(i \neq j)$,

$$\pi_i^{(t+1)} = \frac{i \neq \Sigma \; {}^{j} \, \pi_j^{(t)}}{k-1}$$

and hence

$$\cos \Delta = \frac{1}{\sqrt{k-1}} \, \Sigma \, \pi_i^{(t)}$$

$(1 - \pi_i^{(t)}) \leqslant 1$, equality being reached if $\pi_i^{(t)} = 1/k$ for all i.

Another measure of mobility can be had by comparing the variances of $\pi_j^{(t)}$ s. Now $\pi^{(t)} = \pi^{(t+1)} = 1/k$ and hence

$$\text{Var} \{\pi^{(t)}\} = \Sigma_j \, {\pi_j^{(t)}}^2 - 1/k, \; \text{Var} \{\pi^{(t+1)}\} = \Sigma \, {\pi_j^{(t+1)}}^2 - 1/k.$$

We can consider the rates $R = \text{Var } \pi^{(t+1)} / \text{Var } \pi^{(t)}$ or equivalently the ratio $R' = \Sigma \, \pi_j^2 \, (t+1) / \Sigma \, {\pi_j^{(t)}}^2$. Obviously

$$R' = \frac{\pi^{(t+1)'} \, \pi^{(t+1)}}{\pi^{(t)'} \, \pi^{(t)}} = \frac{\pi^{(t)'} \, P' \, P \, \pi^{(t)}}{\pi^{(t)'} \, \pi^{(t)}} \, .$$

Hence $R' = 1$ if and only if $P'P = I$ or if P is of the form P_1 or a variant of it in respect of row and column permutations only. It has to be noted, however, that this is satisfied if the society is perfectly immobile as also if the society is in a state of extreme movement, characterised by transition probabilities $p_{ii} = 0, p_{ik} = 1$ for some k and $p_{ij} = 0$ $(j \neq k)$. If, however, $p_{ii} = 0$ and $p_{ij} = 1/k-1$ $(i \neq j)$,

$$R' = \frac{\Sigma (1-p)^2}{(k-1)^2 \, \Sigma p_i^2} = \frac{\Sigma q_i^2}{(k-1)^2 \, \Sigma p_i^2}.$$

In case of a perfectly mobile society characterised by $p_{.i} = 1/k$,

$$R' = \frac{1}{k \, \Sigma p_i^2}.$$

It is, however, more meaningful to consider the measure R at least physically. And limits for R attainable in cases of perfect mobility, perfect immobility and extreme movement respectively are 0, 1 and

$$\frac{\Sigma q_i^2/(k-1)^2 - 1/k}{\Sigma p^2 - 1/k}.$$

A Comparison

It is important to consider the problem of estimating the measures of mobility from sample data. Usual data available will give distributions of the total population in successive times (generations) among various social, economic or occupational groups. Transition frequencies will require case histories and will not be generally available. In this context; measures of mobility can be grouped into the following categories ;

(i) measures based on transition probabilities only;
(ii) measures based on transition probabilities and the limiting distribution;
(iii) measures based only on two successive distributions; and
(iv) measures based on transition probabilities and the initial distribution.

Evidently, measures of the third category suggested by the present authors arc at an advantage.

An Illustration

An empirical study of social mobility by Glass and Hall (1954) based on 3,500 pairs of fathers and sons in Britain may be considered to illustrate the various measures referred to in the earlier pages. Actual distributions of fathers and of sons among 7 social groups and the transition probabilities arc as follows :

.388	.146	.202	.062	.140	.047	.015
.107	.267	.227	.120	,206	.053	.020
.035	.101	.188	.191	.357	.067	.061
.021	.039	.112	.212	.430	.124	.062
.009	.024	.075	.123	.473	.171	.125
.000	.013	.041	.088	.391	.312	.155
.000	.008	.036	.083	.364	.235	.275

ACTUAL AND PREDICTED EQUILIBRIUM DISTRIBUTIONS AMONG **SOCIAL** CLASSES

Class	*Fathers*	***Sons***	*Equilibrium*
1. Professional and Higher Administrative	.037	.029	.023
2. Managerial and Executive	.043	.046	.042
3. Higher Grade Supervisory and **Non-Manual**	**.098**	.094	**.088**
4. Lower Grade Supervisory and **Non-Manual**	**.146**	.131	.127
5. Skilled Manual and Routine and Non-Manual	.432	.409	.409
6. Semi-skilled Manual	.131	.170	.182
7. Unskilled Manual	**.111**	.121	.129

Commenting without the aid of any statistical measures of mobility, one can say that there has been an appreciable transition from one class to other classes. There has been an increased proportion of sons in each of classes 2, 6 and 7, particularly in classes 6 and 7 at the cost of a fall in the proportion of individuals in professional and higher administrative services. The lowest two classes to which few fathers belonged have been occupied by a few of their sons. The predicted *equilibrium* distribution shows that the population of *ultimate successors consists* largely of manual workers.

Values of the various measures of mobility calculated from the above data as also their limiting values are presented in Table 1.

TABLE 1

Measure	*Observed here*	*Values of measure observable in cases of*		
		perfect immobility	*perfect mobility*	*extreme movement*
$\|P\|$	.0034	1	0	1/6 = .00002
tr P	2.1140	7	1	0
D	1.0475	0	2.2857	− 2.4602
$1 - \Sigma p_{ii} \pi_i$	.6486	0	0.8571	− 1
Δ^2	.0041	0	.1497	− .4640
R	.8977	1	0	− .0278

It is seen that the measures $|P|$, D, $1 - \Sigma p_{ii} \pi_i$ and Δ^2 do not increase monotonically from perfect immobility through perfect mobility to extreme movement. The other measures of mobility calculated from the given data are shown in Table 2.

TABLE 2

Class (j)	*Measures*				
	μ_j^*	$\frac{p_{jj}^{(2)}}{p_{jj}}$	$\frac{p_{jj}^{(4)}}{p_{jj}}$	$\frac{p_{jj}^{(6)}}{p_{jj}}$	$\frac{p_{jj}^{(8)}}{p_{jj}}$
1	1.5964	.454	.144	.080	.064
2	1.3070	.449	.213	.169	.157
3	1.1232	.628	.495	.473	.468
4	1.1079	.665	.604	.599	.599
5	1.1214	.892	.871	.870	.869
6	1.1890	.689	.603	.590	.587
7	1.1997	.602	.493	.478	.474

Judged by all these measures, the society is somewhat mobile but not near perfectly mobilte, not to speak of extreme movement. Since limits for the different measures corresponding to states of perfect immobility and of perfect mobility are different, we can express the difference between the observed value of a measure and the value attainable in case of perfect immobility as a percentage of the difference between the values attainable for perfect immobility and perfect mobility. These figures are 99.66, 81.43, 45.83, 75.67, 2.74, 10.23 for the measures in the order listed in Table 1. It **is** found that the measure *P* shows the society to be most mobile and the measure *R*, shows the other extreme,

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